Playing with Parity

Flipping cups: *You are given 4 plastic cups. 3 stay up and 1 stays upside down. Can you put them all upside up or upside down by repeatedly turning 2 glasses at once?*

You have now 7 plastic cups. 3 stay up and 4 stay upside down. Can you put them all upside up or upside down by repeatedly turning 2 glasses at once?

You have now n plastic cups. p stay(s) up and q stay(s) upside down, p+q=n. What has to be true about the numbers p and q to allow you put the cups all upside up or upside down by repeatedly turning 2 glasses at once?

Flipping coins: *You have 5 coins and they are all heads-up. Your friend hides the coins and makes 4 flips (flip =turning any coin over once; the same coin can get flipped more than once, as long as there are 4 flips total). Your friend shows you 4 of the coins. Can you predict if the hidden coin is heads-up or heads-down?*

Circle of friends: *Veronica and her friends sit in a circle. It turns out that both neighbors of each child are of the same gender. If there are 4 boys in the circle, how many girls are there?*

How about n boys?

Self-intersecting path: *6 line segments are connected end-to-end, so that they form a path. Is it possible that each of these line segments crosses exactly one of the other segments? How about 7 or 13 line segments?*

Penny and Dime Magic Trick:

(1) Place one coin in each hand.
(2) Multiply the value of the coin in your left hand by *any* odd number.
(3) Multiply the value of the coin in your left hand by *any* even number.
(4) Add together the results from steps (2) and (3).
(5) Predict which hand your friend's penny is in based on their sum.

Problem: Prove that you cannot find 3 odd numbers so that their sum is equal to 100.

Problem: Insert + and - signs in the expression

5 __ 4 __ 3 __ 2 __ 1

in order to make different outcomes. Do you notice any patterns? What explains the patterns you see?

Now we are prepared to solve the troll and wise men puzzle from "A Dozen Problems" session

Troll and wise men: A troll caught one hundred wise men. Very late at night he came to the prison and said: "Tomorrow all of you will line up in the yard, such that the first wise man sees all ninety nine wise men in front of him, the second sees only ninety eight, and so on. The last person sees nobody. Then, my servants will put either red or green hat on each one's head. I will ask each of you starting from the first wise man what color is the hat on your head. If your answer is right you are free to go. If it is wrong I will kill you and feed to my dogs."

The wise men thought the whole night and came up with the scheme to save as many people as possible. How many can they save and how?